



## Choise of Breakwater Type and Optimum Safety Levels

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# CHOICE OF BREAKWATER TYPE AND OPTIMUM SAFETY LEVELS

Hans F. Burcharth<sup>1</sup>

## INTRODUCTION

Breakwaters belong generally to the more expensive part of port and coastal protection structures. The fact that the main function of breakwaters is to provide shelter for wave action defines automatically the two main problems related to breakwater engineering, namely construction in often very harsh environments with under-water implications, and the variability and uncertainty related to the wave loadings during service life.

These two complications make breakwater engineering very challenging and attracts therefore many researchers. However, for lay man and the spouses of the researchers it is a mystery to understand why a mound of rock thrown in the sea can occupy the brain of highly educated coastal engineers for years.

The present paper presents a discussion on the choice of breakwater type and the related optimum safety levels, exemplified by optimum safety levels for conventional rubble mound breakwaters. As a background are presented an overview of design methods and related principles of safety implementation. This includes a discussion of the insufficiency of existing standards with respect to implementation of safety in breakwaters.

## CHOICE OF BREAKWATER TYPE

Design of breakwaters involves the steps indicated in the box diagram, Fig. 1.

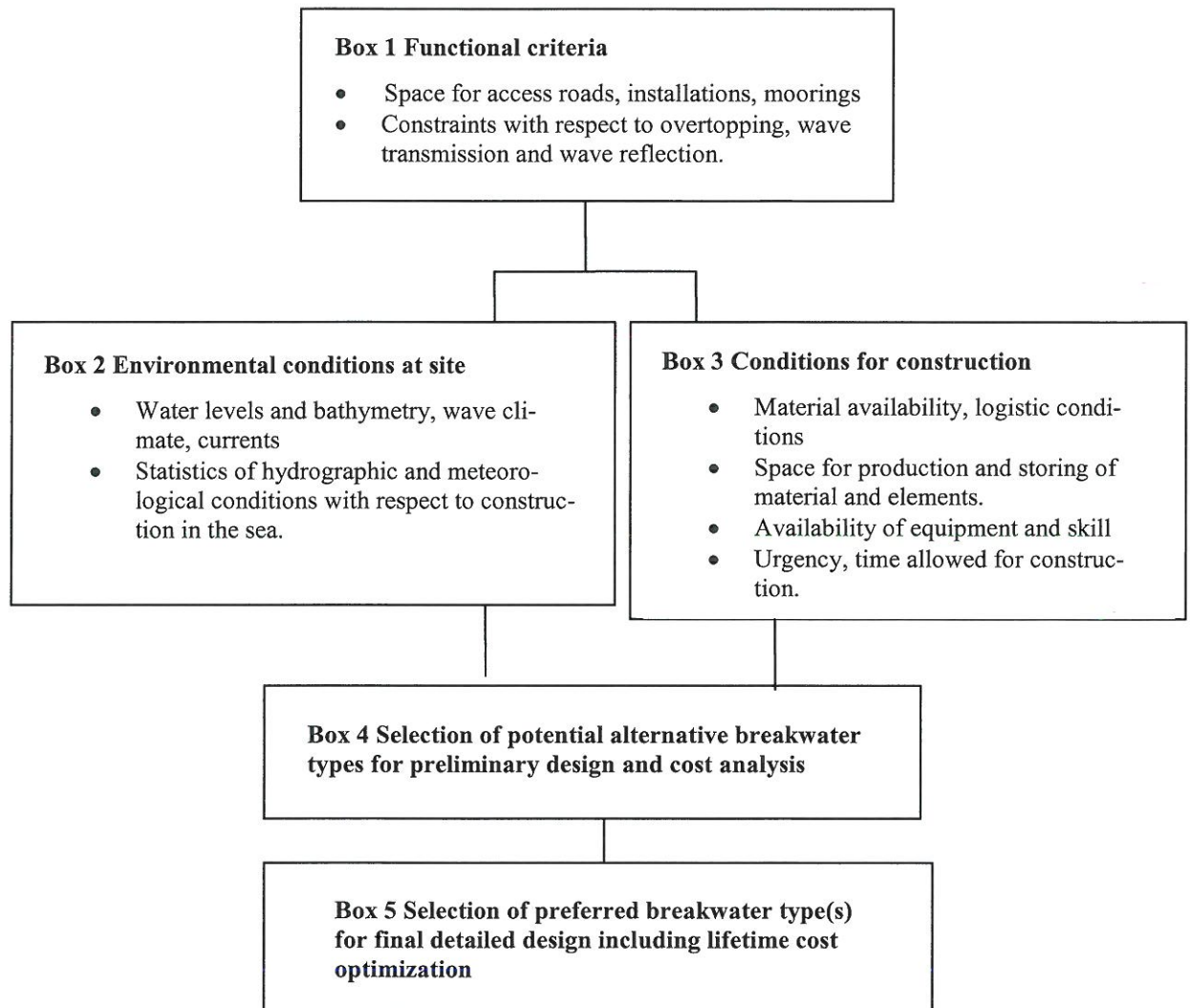


Fig. 1. Steps in selection of type of breakwater

After defining the functional criteria for the structure and after investigation of the environmental conditions and the conditions for construction, one has to select one or more breakwater types for further analyses in terms of conceptual design and costing.

The main types of breakwaters, shown in Fig. 2, have different characteristics, especially with respect to vulnerability to wave overloading and to constructability.

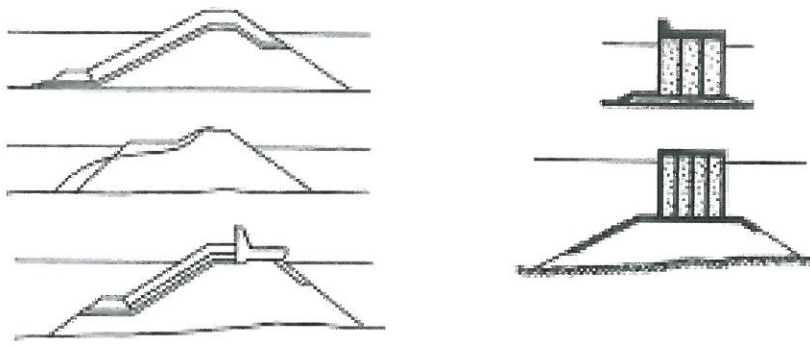


Fig. 2. Main types of breakwaters

Sometimes the choice is very easy. If for example stone materials are easily accessible, water depth is not very large, sea bed soils are relatively weak and soft, restrictions on wave reflection, no need for moorings just behind the breakwater, then the obvious initial choice would be a rubble mound structure.

On the other hand if large quantities of stone material is lacking or transport and dumping cause problems, sea bed soil strength is reasonable good, water depths are medium or larger, significant wave reflection is acceptable, moorings on harbor side are needed, minimum time of construction is needed, caisson can be produced in a sheltered area nearby and be transported and placed in weather windows of sufficient lengths, breaker zones can be avoided, then the obvious initial choice would be a vertical wall caisson structure.

However, the conditions in practice are almost never “clean”, for which reason many alternatives involving both rubble mound and caisson structures can be suitable with respect to function, constructability and environmental impacts. Moreover, modifications as for example replacing weak and soft sea bed soils by better materials can in some cases make a caisson solution just as feasible as a rubble mound solution.

A cost analysis of the various alternatives is generally the basis for the final choice of structure type. A true economical evaluation of alternatives must be based on comparison of total expenses during the service lifetime/design working life.

These expenses depend on the chosen safety level of the structure. The link between safety level and lifetime costs is discussed in the following.

## DESIGN METHODS AND RELATED PRINCIPLES OF SAFETY IMPLEMENTATION

### Deterministic design

Current breakwater design practice makes use of empirical formulae and model tests. When based on national standards or recommendations in which overall safety factors are given, the term *deterministic* design is used. This is to distinguish from *probabilistic* design procedures in which the uncertainties on load and resistance parameters as well as on the design formulae and methods of calculation are taken into account.



The deterministic method prescribe most commonly the use of a specific return period sea state (e.g. 50 years) together with overall safety factors (typically on wave height) and specific values of some structural parameters (e.g. friction between concrete slab and rubble foundation). The method is usually denoted a Level I method. Distinction is made between interim and permanent structures. The prescribed values are determined on the basis of experience with the performance of existing structures.

The actual safeties of the designs are unknown in terms of probability of damage or failure within the lifetime of the structure.

### Design based on conventional partial safety factors

The partial safety factor system is applied in the EURO NORM (EN). As an illustrative example of the concept of using partial safety factors is considered the limit state equation for horizontal sliding of a caisson on a rubble foundation.

$$(F_G - F_U) f - F_H \begin{cases} \geq 0, \text{ no sliding} \\ < 0, \text{ sliding} \end{cases} \quad (1)$$

where

$F_G$  = Buoyancy reduced weight of the caisson

$F_U$  = Wave induced uplift force

$F_H$  = wave induced horizontal force

$f$  = Friction coefficient for base plate on rubble stone foundation

The variables in the limit state equations are either load variables,  $X_i^{load}$  as for example  $F_H$  and  $F_U$ , or resistance variables,  $X_i^{res}$  as for example  $F_G$  and  $f$ .

The variables are uncertain parameters. Therefore are applied partial safety factors,  $\gamma_i$  to characteristic values of each parameter, or if sufficient, to some of them in order to obtain the design values:

$$X_i^{design} = \gamma_i^{load} \cdot X_{i,ch}^{load} \quad (2)$$

$$X_i^{design} = \frac{X_{i,ch}^{res}}{\gamma_i^{res}}$$

The partial safety factors,  $\gamma_i$ , which are always larger or equal to one, are uniquely related to the definition of the characteristic values of the uncertain parameters. In conventional civil engineering codes, the characteristic values of material strength parameters are taken as the lower 5 % fractile, while for load parameters characteristic values corresponding to the upper 5% fractile are often used. Other definitions may be used, as is the case in the PIANC WG 12 and WG 28 system. The magnitude of  $\gamma_i$  reflects both the target safety level, the uncertainty on the related parameter  $X_i$ , and the relative importance of  $X_i$  in the failure mode equation.

When the partial safety factors and the characteristic values of the parameters are applied in the limit state failure mode equation, we obtain a *design equation* which in its general formulation for a sliding failure reads:

$$G = \left( \frac{F_G^{ch}}{\gamma_{FG}} - \gamma_{FU} \cdot F_U^{ch} \right) \frac{f}{\gamma_f} - \gamma_{FH} F_H^{ch} \geq 0 \quad (3)$$

The partial safety factors are calibrated such that when applied in the design of conventional structures the outcome will be structures with safety levels close to target safety levels. The target safety levels correspond to experienced and accepted satisfactory performance of the specific structure type, for example conventional buildings. Although the partial safety factors are calibrated to specific target safety levels for some standard structures, application will generally not result in structures having the target safety levels but safety levels in the same order of magnitude.

The safety levels for load bearing conventional civil engineering structures such as tall buildings and bridges are traditionally much higher than for breakwaters.

EURO NORM does not yet include partial safety factors suitable for coastal structure design.

### **The PIANC partial safety factor system for breakwaters**

A new type partial safety factors system was developed In the PIANC Working Group 12 on rubble mound breakwaters. The safety factors on load and resistance are in this system given as function of the target safety level chosen for the specific structure in terms of probability of damage within the defined working life of the structure (PIANC 1992b, PIANC 2003, Burcharth and Sorensen 2000). The system, which covers most failure modes for conventional rubble mound and caisson breakwaters facilitate easy design to a specific safety level. The PIANC partial coefficient safety factors cover the range of probability of failure (damage) within working life from 0.01 to 0.4. However, the system has not been widely used, most probably because no target safety levels in terms of probability of damage within working life have been given in national standards and design recommendations. One of the main objectives of the ongoing PIANC Working Group 47 is to analyse and propose economically optimized safety levels. With such levels in hand the PIANC system can be applied directly for design. The system can also be used for easy evaluation of the safety of a given design. The method corresponds to Level II.

The optimum safety levels will be published in a PIANC Working Group 47 report. Typical results for conventional rubble mound breakwaters are presented in this paper.

### **Probabilistic design**

The basic principle in probabilistic design is to take into account all uncertainties related to loadings, and structural strength as well as uncertainties related to design tools like formulae, computational methods and physical model tests. All uncertain parameters are modelled as stochastic variables with assigned probability distributions. This includes the parameters defining the long-term wave statistics.

For design the method involves a trial and error procedure in which the reliability/safety of a proposed design is estimated by a probabilistic method. If different from the target safety level the design is changed and a new reliability analysis performed, etc. The target reliability is given in terms of probabilities of damage within the design working life in accordance with the performance criteria.



Various techniques can be used for the reliability analysis. The safety index method (Level II) implies that all parameter distributions are transformed into Normal-distributions. Monte Carlo simulations in which the actual parameter distributions are applied (Level III) are commonly used today because of the development in computer capacity. This method is used in the optimum safety level computations presented in this paper.

## **SAFETY CLASSIFICATION, WORKING LIFE AND SAFETY LEVELS IN RECENT CODES AND RECOMMENDATIONS**

Uncertainties related to environmental conditions, design formulae and construction accuracy make it necessary to implement a safety margin in the design (PIANC 1992b). The margin depends on the economic, environmental and social importance of the structure. This leads to a definition of safety classes, design working life and related recommended safety levels.

The Euro Norm EN 1990 and the ISO 2394 standard define the design working life as the assumed period for which a structure is to be used for its intended purpose, given appropriate maintenance. Table 1 shows the indicative/example values of design working life given in ISO 2394.

**Table 1. Proposed classification of design working life, (ISO 2394).**

<b>Class</b>	<b>Notional design working life (years)</b>	<b>Examples</b>
1	1 to 5	Temporary structures
2	25	Replacement structural parts, e.g. gantry girders, bearings
3	50	Buildings and other common structures, other than those listed below
4	100 or more	Monumental buildings and other special or important structures. Large bridges.

The EN 1990 define almost the same working life except that working life for temporary structures is 10 years and a class for agriculture and similar structures is added with 15-30 years working life.

The actual observed service life for breakwaters is in the range 25-50 years.

The EN 1990 defines the safety classes (denoted consequence classes) shown in Table 2.

**Table 2. Definition of consequences classes (EN 1990:2002).**

Consequences class	Description	Examples of buildings and civil engineering works
CC3	<b>High</b> consequence for loss of human life, or economic, social <i>or</i> environmental consequences <b>very great</b>	Grandstands, public buildings where consequences of failure are high (e.g. a concert hall)
CC2	<b>Medium</b> consequence for loss of human life, economic, social or environmental consequences <b>considerable</b>	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building)
CC1	<b>Low</b> consequence for loss of human life, <i>and</i> economic, social or environmental consequences <b>small or negligible</b>	Agricultural buildings where people do not normally enter (e.g. storage buildings), green-houses

Corresponding to the classes given in Table 2 are defined the recommended minimum reliabilities and related failure probabilities,  $P_f$ , shown in Table 3.

**Table 3. Recommended minimum values of reliability index  $\beta$  and related failure probabilities  $P_f$  for construction works (ultimate limit states), (EN 1990:2002).**

Reliability Class	Minimum values for $\beta$	
	1 year reference period	50 years reference period
RC3	5,2 ( $P_f = 10^{-7}$ )	4,3 ( $P_f = \text{app} \cdot 10^{-5}$ )
RC2	4,7 ( $P_f = 10^{-6}$ )	3,8 ( $P_f = \text{app} \cdot 10^{-4}$ )
RC1	4,2 ( $P_f = 10^{-5}$ )	3,3 ( $P_f = \text{app} \cdot 5 \cdot 10^{-4}$ )

If the classification in Table 2 has to be used for breakwaters then class CC1 would be the most relevant although really not suitable. The related max. probability of failure of  $P_f = \text{app} \cdot 5 \cdot 10^{-4}$  for a 50 years reference period, given for class RC1 in Table 2, is much too conservative for breakwaters and therefore not applicable.

## LIMIT STATE DESIGN

So-called performance based design has in recent years been presented by some coastal engineers as something new. This is of course not the case. Ever since design formulae like the armour stability formulae of Iribarren and Hudson have been available it was necessary for



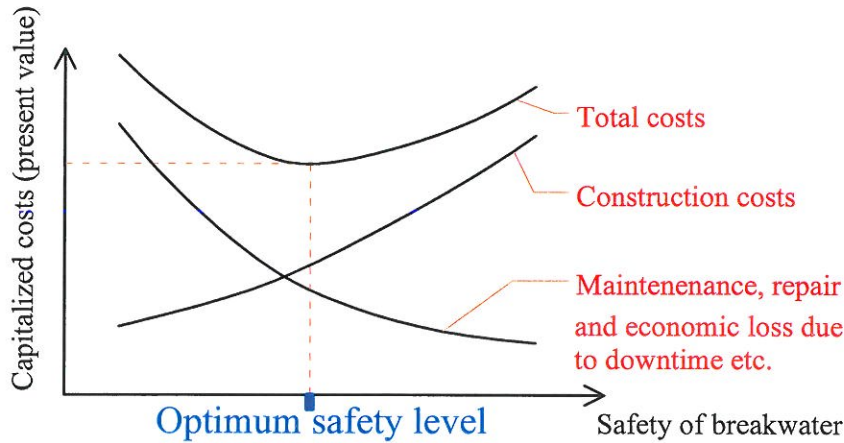
the designer to choose a damage level (in terms of stability coefficient) and a return period design wave. This inherently defines a target performance, but only for one design state. The new approach is that more design stages are considered, as is also requested in the ISO-standard 2394 on Reliability of Structures. In this document is stated that both a serviceability limit state (SLS) and an ultimate limit state (ULS) must be considered, and damage criteria (performance criteria) must be assigned to these limit states. Moreover, uncertainties on all parameters and models must be taken into account in the design.

For breakwaters it is relevant to introduce also a repairable limit state (RLS) defined as the limit state where repair by foreseen method can be applied. For example a stage of damage which still allows access of a crane on the crest of the structure.

## SAFETY LEVELS BASED ON COST OPTIMIZATION

In the following is given a short presentation of the cost optimization analyses performed for the PIANC WG 47 by the author in cooperation with Prof. John Dalsgaard Sorensen, Aalborg University.

The idea of the analyses is explained by Fig. 3 which illustrates the objective of identifying the safety level related to the minimum total cost over the service life. This includes capital costs, maintenance and repair costs as well as downtime costs.



**Fig.3. Illustration of identification of safety level corresponding to minimum costs in structure service life.**

All costs are discounted back to the time of construction of the breakwater by the use of eq. (4).

$$\min_T C(T) = C_I(T) + \sum_{t=1}^{T_L} \left\{ C_{R_1}(T)P_{R_1}(t) + C_{R_2}(T)P_{R_2}(t) + C_F(T)P_F(t) \right\} \frac{1}{(1+r)^t} \quad (4)$$

where

$T$  return period used for deterministic design

$T_L$  design life time

$C_I(T)$  initial costs (building costs)

$C_{R_1}(T)$  cost of repair for minor damage when SLS is exceeded

$P_{R_1}(t)$  probability of minor damage in year  $t$

$C_{R_2}(T)$  cost of repair for major damage when RLS is exceeded

$P_{R_2}(t)$  probability of major damage in year  $t$

$C_F(T)$  cost of failure including downtime costs when ULS is exceeded

$P_F(t)$  probability of failure  $t$

$r$  real rate of interest

The procedure in the optimization calculations is as follows:

1. Select type of breakwater, water depth and long-term wave statistics.
2. Extract design values of significant wave height  $(H_S^T)$  and wave steepness corresponding to a number of return periods,  $T = 5, 10, 25, 50, 100, 200$  and 400 years.
3. Select service lifetime for the structure, e.g.  $T_L = 25, 50$  and 100 years.
4. Design by conventional deterministic methods the structure geometries corresponding to the chosen  $H_S^T$  – values.
5. Calculate construction costs for each structure.
6. Define repair policy and related cost of repair.
7. Specify downtime costs related to damage levels.
8. Define a model for damage accumulation.
9. For each structure geometry use stochastic models for wave climate and structure response (damage) in Monte Carlo simulation of occurrence of damage within service life time. The structures are exposed to storms corresponding to real long-term statistics occurring in accordance with a Poisson process.
10. Calculate for each structure geometry the total capitalized lifetime costs for each simulation. Calculate the mean value and the related safety levels corresponding to defined design limit states.
11. Identify the structure safety level corresponding to the minimum total costs.

The downtime costs are set to a loss of 200,000 Euro per day over a period of 90 days, i.e. 18,000,000 Euro. This cost, which corresponds to direct loss if a large container berth closes down, is related to a breach in a 1 km stretch of the breakwater.

The failure modes which are considered by the PIANC Working Group 47 in their analyses are the most important for the chosen types of structures.

Rubble mounds:	Displacement of blocks in main armour
Berm breakwaters:	Recession of seaward berm shoulder
Caisson breakwaters on bedding layer and high rubble foundation:	Horizontal sliding of caisson Slip failure in rubble foundation Slip failure in seabed soil

Only the main failure modes are taken into account. Inclusion of more but less important failure modes will not change the optimum safety levels related to the main failure mechanisms.

Moreover, the extra construction costs of strengthening secondary structure elements to a degree of negligible failure probability are very small. This explains why correlation (interaction) between main failure modes and other failure modes is not included in the simulations.

In this paper is presented results solely for the conventional rubble mound breakwater.

The applied stochastic modelling of all parameters and design formulae used in the Monte Carlo simulations is based on the uncertainties given in the PIANC WG 12 and WG 28 report as well as in the CEM (2006).

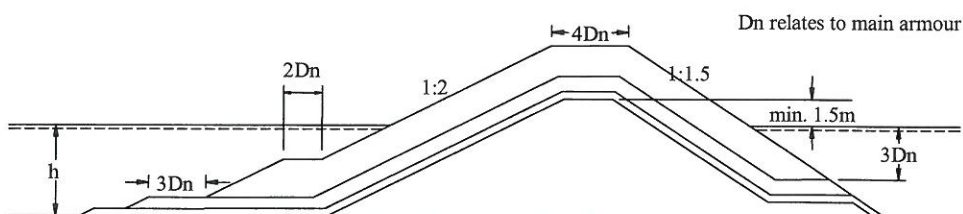
The long-term wave statistics applied in the simulations are from Follonica in Italy (shallow water), Bilbao in Spain, Sines in Portugal and the Baltic Sea. The statistics represent a range of the 100 years return periods significant wave heights from 5.6 m to 13.2 m.

The simulations cover only breakwaters with no berths on the rear side, i.e. some overtopping and related wave transmission can be tolerated.

If only very limited overtopping is allowed then the structure must have higher crest level, but the optimum safety level will hardly change.

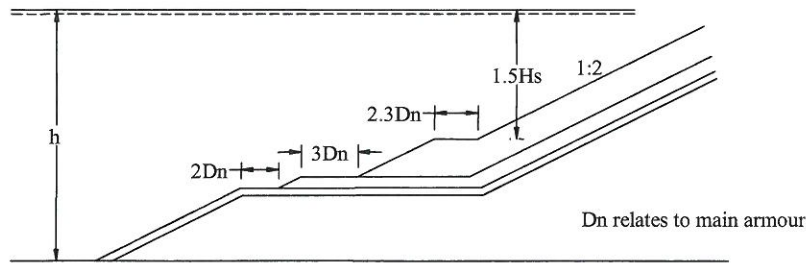
### Optimum safety levels for conventional rubble mound breakwaters

Fig.4 shows the parameterised cross section.



Shallow water cross section:  $h < 1.5 H_S + 2.7 \cdot D_n$





Deep water cross section:  $h = 1.5 H_s + 2.7 D_n$

Fig. 4. Shallow and deep water cross sections

Table 4 presents the limit state armour layer damages in terms of percentage of displaced armour units as well as the related repair policy.

**Table 4. Limit state damages and repair policy**

Limit state	Armour damage D	Repair policy
SLS	5%	Repair of armour
RLS	15%	Repair of armour and filter 1
ULS	30%	Repair of armour, filter 1 and filter 2

The construction built-in unit prices for medium size breakwaters in 10-15 m water depth are 10, 16, 20 and 40 EURO/m<sup>3</sup> for core, filter 1, filter 2 and armour respectively. For very large structures in deeper water the prices are reduced.

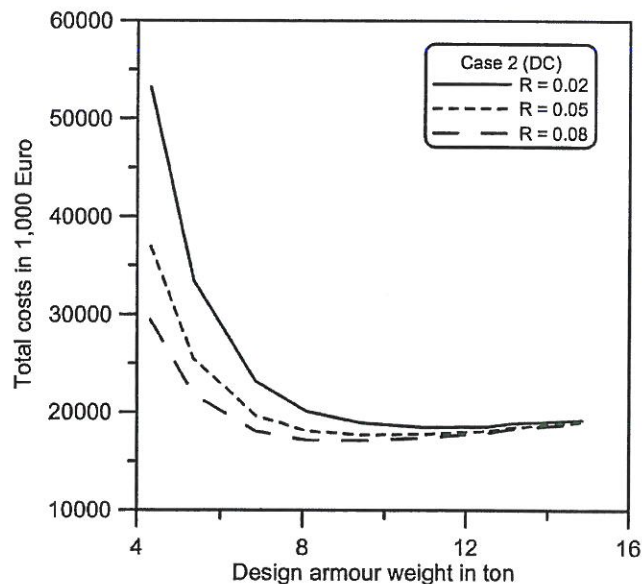
The unit prices for repair are set 50% higher than the construction unit prices. Moreover is added 33% to the repair of armour costs in order to cover mobilisation costs. It is assumed in the simulations that repairs are completed before a damaging storm arrives. This is somewhat on the unsafe side.

Table 5 shows a typical example of the outcome of the simulations for conventional rubble mound breakwaters. The case is a concrete cube armoured breakwater in 15 m water depth. The service life is 50 years. Values are given for interest rates 2%, 5% and 8%. Damage accumulations are included in the simulations. The van der Meer (1988) cube stability formula is used.

**Table 5. Example of optimum safety levels for cube armoured breakwater in 15 m water depth. 50 years service life. Slope 1:2. Wave steepness  $s_{om} = 0.025$**

Real interest rate (%)	Downtime Costs	Deterministic design data			Optimum limit state average number of events within service lifetime			Initial costs (1,000 EURO)	Total costs (1,000 EURO)
		Optimum design return period, T yrs	$H_s^T$ (m)	Armor unit mass, W (t)	SLS	RLS	ULS		
2	None	100	5.64	9.45	3.35	0.06	0.02	16038	18029
5		50	5.36	8.09	5.31	0.11	0.04	15316	17094
8		50	5.36	8.09	5.31	0.11	0.04	15316	16495
2	Included	200	5.92	10.93	2.13	0.03	0.01	16763	18498
5		100	5.64	9.45	3.35	0.06	0.02	16038	17694
8		100	5.64	9.45	3.35	0.06	0.02	16038	17140

Fig. 5 shows the variation in lifetime costs with structural safety in terms of armour unit mass in tons.



**Fig. 5. Lifetime costs as function of interest rate and concrete cube armour unit mass used in deterministic design. Damage accumulation and downtime costs included.**

The graphs in Fig. 5 show fairly flat minima. This means that a conservative design can be chosen without causing extra lifetime costs. This is characteristic for conventional rubble mound structures and is caused by their ductile damage development.

For rubble mound structures armoured with complex type of interlocking blocks (e.g. Accropodes and Xblocks) on steep slopes there is a more distinct minimas due to the brittle failure development. Such distinct minimas are also characteristic for caissons which are not allowed to slide or where a foundation slip failure is the critical failure mode. If caissons are allowed to slide to some extent and foundation failures are not the critical failure mode then there will be a ductile failure development and very flat minimas.

From table 5 it is seen that optimum safety level corresponds – if downtime is included and interest rate is 2% - to two SLS repairs, 3% probability of reaching the RLS, and 1% probability of reaching the ULS all in 50 years lifetime. However, a more conservative design corresponding to one SLS repair in the lifetime will not be significantly more expensive.

## EXAMPLE OF DESIGN USING THE PIANC PARTIAL SAFETY FACTORS

The partial safety factors corresponding to concrete cube armour designed by the van der Meer (1988) formula are given for failure probabilities 1%, 5%, 10%, 20% and 40% in Table 6. The safety factors are taken from the PIANC WG 12 report (1992) and from Burcharth and Sørensen (2000).

The implementation of the partial safety factors in the van der Meer (1988) cube stability formula is given in eq. (5). The original formula valid for slope 1:1.5 has been modified to cover slope  $\cot \alpha$

**Table 6. Partial safety factors for stability of cube armour for conditions of no model tests performed. PIANC WG 12 (1992) and Burcharth and Sorensen (2000).**

	good quality wave data $\sigma' F_{H_s} = 0.05$		poor quality wave data $\sigma' F_{H_s} = 0.2$	
	$\gamma_H$	$\gamma_Z$	$\gamma_H$	$\gamma_Z$
$P_f$				
0.01	1.5	1.10	1.8	1.04
0.05	1.3	1.08	1.5	1.04
0.10	1.3	1.00	1.4	1.02
0.20	1.2	1.00	1.2	1.06
0.40	1.0	1.08	1.0	1.10

$$\frac{\gamma_H H_s^T}{\Delta D_n \gamma_Z} = \frac{1}{\gamma_Z} \left( 6.7 \frac{N_{od}^{0.4}}{N_z^{0.3}} + 1.0 \right) s_{om}^{-0.1} \left( \frac{\cot \alpha}{1.5} \right)^{1/3} \quad (5)$$



where

$\gamma_H$ and $\gamma_Z$	Partial safety factors, see Table 6
$H_s^T$	central estimate of $H_s$ corresponding to return period $T$ = service life of structure
$N_{od}$	damage parameter
$N_Z$	number of waves
$s_{om}$	deep water wave steepness
$\Delta$	$\rho_s / \rho_w - 1$ , $\rho_s$ and $\rho_w$ are mass density of cube and water respectively
$D_n$	equivalent cube side length

Consider a concrete cube armoured breakwater with slope 1:2 in 15 m water depth to be designed for a service lifetime of  $T = 50$  years. No downtime costs are anticipated in case of damage. Real interest rate is assumed to be 5% p.a.

Design wave conditions are

$$H_s^T = H_s^{50 \text{ years}} = 5.36 \text{ m, good quality wave data } \sigma'_{F_{HS}} = 0.05$$

$$s_{om} = 0.025$$

$$N_Z = 1,000$$

$$\Delta = 1.34$$

The optimum safety levels in terms of limit state average number of events within service lifetime (from table 5) and the related safety factors  $\gamma_H$  and  $\gamma_Z$  (inter- and extrapolated from Table 6) are shown in Table 7 together with optimum cube mass  $W$ , calculated from (5).

**Table 7. Optimum mass of cubes**

Limit state	Opt. no of events	$\gamma_H \gamma_Z$	Cube mass $W$ (t)
SLS	5.31	1.05	7.9
RLS	0.11	1.28	8.5
ULS	0.04	1.44	9.3

In principle should  $W$  be the same for all limit states. This is however not completely possible and not important due to the characteristic flat minima shown in Fig. 5.

The agreement with the min.  $W$  of 8.09 t given in Table 5 is very good.

The example demonstrates the ability of the PIANC partial safety factor system to design for a target reliability, in this case the reliability corresponding to minimum lifetime costs of the breakwater.

It should be noted that in case of depth limited waves it is not possible to use the partial safety factors given in the tables in Burcharth and Sorensen (2000). In such cases it is necessary to calculate the partial safety factors by the formulae given in the same reference.

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